

On the multiplicity distributions at LHC energies

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Abstract

The ALICE and CMS data on the multiplicity distributions are compared with the lower energy data and with the results from the 8.142 version of the PYTHIA MC event generator with two tunings. The ALICE data for moments are used to calculate the factorial cumulants. It is suggested that the data on moments or cumulants are well suited to specify the optimal tuning of the model parameters.

PACS: 13.85.-t, 13.90.+i

Keywords: LHC, multiplicity distributions

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1 Introduction

With the advent of LHC data it became possible to investigate the multiplicity distributions at the CM energies beyond 2 TeV. It is very interesting to check how well the default versions of MC generators describe the minimum bias events in this energy range. In particular, it is tempting to look for a best tuning of the model parameters using only the data from the multiplicity distributions.

In a recent note [1] we have discussed the energy dependence of the central density (defined by the average charged multiplicity in a central bin in pseudorapidity). We have shown that, contrary to some claims, the fast increase with energy observed in the ALICE data [2] is not unexpected. In fact, we found that the default version of PYTHIA 8.135 generator [3], [4] predicts a too fast increase, but with some tuning the data may be well described. The moments for three selected pseudorapidity bins were also compared with data [5]. The qualitative agreement was observed and the spread of results for two different tunings was surprisingly small.

In this paper we discuss the data on the multiplicity distributions listed above as well as the data from the CMS experiment [6] compared with some lower energy data [7], [8], [9]. We use here the new version of PYTHIA 8.142 [10] with the default tuning and with the tuning used by default in the older version 8.135. Let us note that the PYTHIA 8.142 version is significantly changed compared to its earlier versions (PYTHIA 8.107 and 8.135) which we have used in a previous publication [1]. The changes, concerning mainly the final state radiation, are motivated by the discrepancy between the Tevatron data for "underlying event" and the model results. The qualitative conclusions of our note [1] are, however, unchanged.

In the next section, we compile for convenience the formulae defining the moments, factorial moments and factorial cumulants of the multiplicity distributions. Then we recall the results from Ref. [1] and compare them with the factorial cumulants calculated from the data and MC generators. We will see that that the scaled factorial cumulants K_q , calculated from the published values of the standard scaled moments c_q and the average multiplicities \bar{n} , exhibit much smaller spread than suggested by the published values of the uncertainties of c_q and \bar{n} . Thus it would be more reasonable to use the scaled factorial cumulants of the multiplicity distributions to test the specific models of the high energy collisions. Finally, we discuss the energy dependence of the average multiplicity and second moment in a wider energy range, as presented in the CMS paper [6]. The last section contains some conclusions and the outlook.

2 Moments and cumulants

The multiplicity distributions are often parametrized in terms of moments. This facilitates the comparison with models and allows for a simple description of the energy dependence. The crucial problem is the proper choice of the set of moments to be used. A standard first choice is to use simple power moments defined by

$$\bar{n}^q = \sum n^q P(n)$$

or their scaled version

$$c_q = \frac{\bar{n}^q}{\bar{n}^q}.$$

These moments are easy to calculate and (for moderate q) depend quite uniformly on the probabilities, although obviously the lowest multiplicities are suppressed, and the

high multiplicity tail is enhanced. The use of \bar{n} and a few lowest c_q moments allows to parametrize the multiplicity distribution quite satisfactorily. In the high energy (high \bar{n}) limit these moments allow to describe the "KNO scaling function" $\Psi(x)$ [11] defined by

$$\Psi(x) = \lim_{\bar{n} \rightarrow \infty} P(n)/\bar{n}$$

where $x = n/\bar{n}$. Obviously, in this limit

$$c_q = \int z^q \Psi(z) dz.$$

However, for the distributions in small phase space bins the power in the denominator of the formula for the scaled moments results in the development of trivial singularities. Thus since some time one uses more often the factorial moments. If we define the factorial quotient

$$n_q = \frac{n!}{(n-q)!}$$

the corresponding standard and scaled factorial moments are, respectively

$$\bar{n}_q = \sum n_q P(n)$$

and

$$F_q = \frac{\bar{n}_q}{\bar{n}^q}.$$

The factorial moments of the order q are the integrals of the q -particle densities for identical particles. Obviously, $\bar{n}_1 = \bar{n}$ is the integral of the single particle density. For the smooth phase space distributions the scaled factorial moments behave smoothly for the bin size decreasing to zero, and the possible power increase is a signal for intermittency [12]. However, the drawback of the definition of the higher factorial moments is their independence on the lower end of the multiplicity distribution. Moreover, the scaled factorial moments of different order are strongly correlated.

Therefore it is preferable to parametrize the multiplicity distributions by the factorial cumulants. They are defined in a compact way by the generating function

$$G(z) = \sum z^n P(n).$$

The factorial cumulants f_q (called also "Mueller coefficients" [13]) are defined by

$$f_q = \left. \frac{d^q (\ln G(z))}{dz^q} \right|_{z=1}$$

to be compared with an analogous definition of the factorial moments

$$\bar{n}_q = \left. \frac{d^q G(z)}{dz^q} \right|_{z=1}.$$

The scaled factorial moments are defined in a usual way

$$K_q = \frac{f_q}{\bar{n}^q}.$$

By definition, for $q = 1$ all the scaled moments c_q , F_q and K_q are equal one. Another name for the factorial cumulants f_q is "the correlation integrals", as they are the integrals of

the correlation functions of the order q . Therefore they measure the genuine multiparticle correlations. For the uncorrelated emission all the factorial cumulants for $q > 1$ vanish. If there are only two-particle correlations, $f_q = 0$ for $q > 2$. The values of the factorial cumulants of different orders are uncorrelated.

One may add that another set of moments was advocated [14]

$$H_q = K_q/F_q = f_q/\overline{n}_q.$$

These moments were shown to have strongly reduced statistical uncertainties even for the order up to $q = 10$. However, for $q < 5$ they are not very practical to use, as their values quickly decrease with increasing q .

A practical difficulty in using the factorial cumulants is the complexity of the formulae for their errors, or, more precisely, for their statistical uncertainties. For the average multiplicity one uses a simple estimate of the uncertainty

$$\Delta\bar{n} = D/\sqrt{N}$$

where D is the dispersion, and N is the total number of measured events. Analogous simple formulae exist for the higher standard moments \overline{n}^q . However, for the scaled moments, and especially for cumulants, the corresponding formulae are more complicated. Moreover, usually they overestimate significantly the observed spread of experimental results.

There is a simple explanation of this fact. The formula for $\Delta\bar{n}$ was obtained from a simple prescription for the statistical uncertainty of a parameter

$$\Delta A = \sqrt{\sum (\partial A / \partial N_n)^2 \cdot (\Delta N_n)^2}$$

with $\Delta N_n = \sqrt{N_n}$. This prescription results from the assumption that the measured numbers of events with different multiplicities N_n are uncorrelated, and their errors are purely statistical. These assumptions were reasonable e.g. for the hydrogen bubble chamber experiments, where the full solid angle was available for the measurements of tracks, and the multiplicity of charged particles (always even for charged beam) was unambiguously measured. This is certainly not the case for a colliding beam experiment with electronic detectors, where the multiplicity distribution is measured in the restricted bin of phase space. The uncertainty of a measurement of the variables defining this bin as well as the effects of the track splitting and joining due to the imperfection of the detector result in non-statistical errors and in the strong correlations between the numbers of events with different multiplicities. Neither a simple formula for $\Delta\bar{n}$ presented above, nor the complicated formulae for the uncertainties of the scaled factorial cumulants (derived from the same prescription for the statistical uncertainties of the numbers of events) are reliable.

Therefore the realistic estimate of the uncertainties of the parameters of the multiplicity distribution requires the full knowledge of the detector and, in particular, the measurement of the correlation matrix for the multiplicities. This can be done only by the authors of the experiment. Readers cannot translate them reliably into a different set of parameters, since their uncertainties will be unknown.

3 Moments and factorial cumulants

In a recent paper [5] the ALICE collaboration has presented the values of average multiplicities and scaled moments c_q for $q = 2, 3, 4$ at two CM energies 0.9 and 2.36 TeV for three choices of the central pseudorapidity bin widths: $\Delta\eta < 1$, $\Delta\eta < 2$ and $\Delta\eta < 2.6$.

In Table 1 and in Fig.1 we show the experimental values of the \bar{n} from the "non-single-diffractive" (NSD) ALICE data at 900 GeV and 2.36 TeV and the corresponding values calculated from the PYTHIA 8.142 default version. For each point we have generated 10^5 events. In all tables the numbers in parentheses denote the statistical and systematic errors.

Table 1: Average multiplicities for three choices of rapidity bins from ALICE and two versions of PYTHIA 8.142 at 0.9 and 2.36 TeV.

η range	ALICE 0.9 TeV	P8.142	P8.142/135	ALICE 2.36 TeV	P8.142	P8.142/135
$ \eta < 0.5$	3.60(2)(11)	3.57	3.87	4.47(3)(10)	4.45	4.75
$ \eta < 1.0$	7.38(3)(17)	7.27	7.88	9.08(6)(29)	9.04	9.66
$ \eta < 1.3$	9.73(12)(19)	9.57	10.35	11.86(22)(45)	11.89	12.68

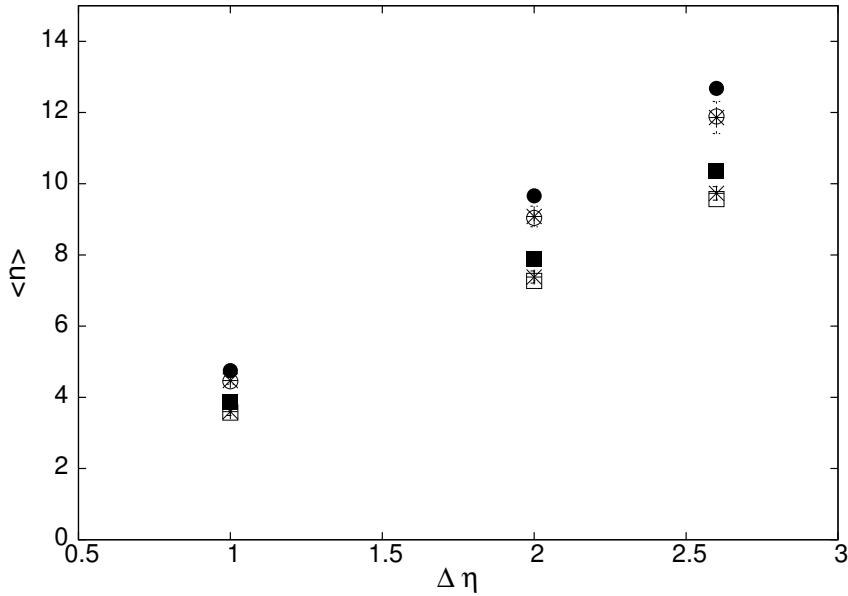


Figure 1: The average multiplicity from the ALICE data at 0.9 and 2.36 TeV (asterisks with error bars), from PYTHIA 8.142 (open squares and circles) and from PYTHIA 8.142/135 (full squares and circles) as a function of the pseudorapidity bin width

The data for the average multiplicities agree well with the model (remember that we are using the default version of PYTHIA 8.142 without any tuning). The agreement is significantly better than that for the central densities of charged particles for inelastic events with at least one particle in the central bin, measured by ALICE at 0.9, 2.36 and 7 TeV [2], although in this case the increase with energy is also reasonably well described, as seen in Table 2 and in Fig.2.

We have repeated the same calculations for the PYTHIA 8.142 with a different set of the model parameter values: the default values from the PYTHIA 8.135 version are taken. Let us remind here that the tuned parameters refer to the formulae used in the description of multiple scattering. The regularization of the (divergent) QCD cross section is done by the introduction of a factor

$$F(p_T) = \frac{p_T^4}{(p_{T0}^2 + p_T^2)^2}$$

Table 2: Central density: data and the results for two versions of PYTHIA 8.142

Energy (TeV)	ALICE	PYTHIA 8.142 d	PYTHIA 8.142/135d
0.90	3.81(1)(7)	3.58	3.86
2.36	4.70(1)(11)	4.41	4.69
7.00	6.01(1)(20)	5.80	6.14

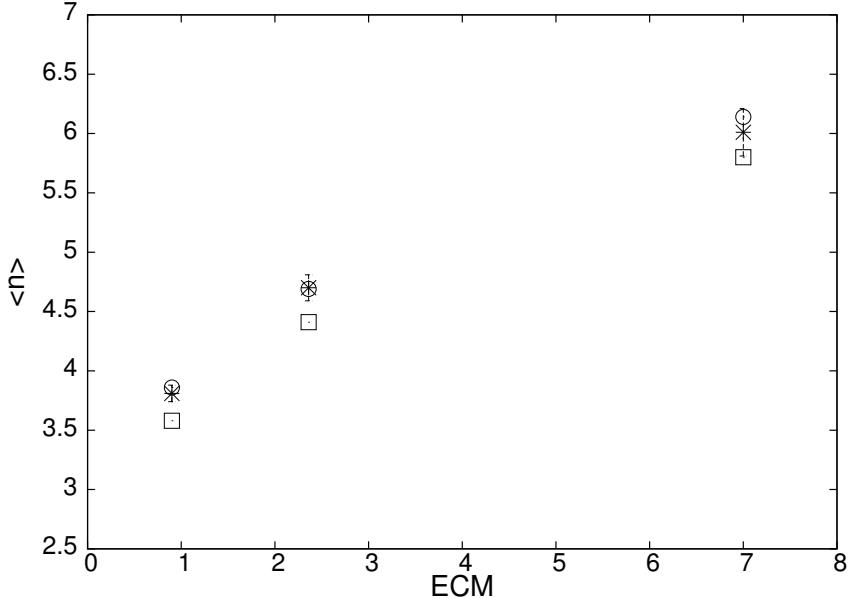


Figure 2: Central density: data (asterisks with error bars), PYTHIA 8.142 default (squares) and PYTHIA 8.142 with 8.135 tuning (circles) as a function of the CM energy.

where

$$pT0 = pT0Ref \left(\frac{ecmNow}{ecmRef} \right)^{ecmPow}$$

and *ecmNow* is the CM energy in GeV. The default values in PYTHIA 8.135 are 2.0 for the *pT0Ref*, 1960.0 for *ecmRef* and 0.16 for *ecmPow*. In PYTHIA 8.142 the corresponding values are 2.15, 1800.0 and 0.24. Moreover, the default version of PYTHIA 8.142 uses a simplified profile of the parton density in the impact parameter given by a Gaussian curve, whereas the standard earlier versions were using two Gaussians (with two extra parameters for the ratios of their slopes and weights). We have found that in this case the results for the average multiplicities are reversed: there is a perfect agreement for central densities in the "INEL>0" sample (see Table 2) and a slight overestimation for the NSD sample.

The difference between Tables 1 and 2 shows that the ALICE procedures give for the PYTHIA events practically the same average multiplicity in the "NSD" and "INEL>0" samples, whereas experimentally the second sample has higher average multiplicity, which suggests lower contribution from the diffractive events. Remember that for PYTHIA we use the same definition of "NSD" and "INEL>0" events as in the data. This means that we generate all the classes of events (non-diffractive, single diffractive and double diffractive) and then remove the events which do not satisfy the conditions defined in the ALICE procedures.

The formulae listed in the previous section allow to express the scaled factorial cumulants in terms of the scaled moments and average multiplicity. For the lowest values of q we have

$$K_2 = F_2 - 1, \quad F_2 = c_2 - 1/\bar{n}, \quad K_3 = F_3 - 3F_2 + 2, \quad F_3 = c_3 - c_2/\bar{n} + 2/\bar{n}^2$$

and

$$K_4 = F_4 - 4F_3 - 3F_2^2 + 12F_2 - 6, \quad F_4 = c_4 - 6c_3/\bar{n} + 11c_2/\bar{n}^2 - 6/\bar{n}^3.$$

For the higher values of q it is more practical to use a recurrence formula expressing K_q by F_q and the K moments of the lower order

$$K_q = F_q - \sum_i \frac{(q-1)!}{(i-1)!(q-i)!} K_{q-i} F_i.$$

In Table 3 we show the values of the c_q and K_q moments at 900 GeV, and in Table 4 the same results at 2.36 TeV.

Table 3: Scaled moments and factorial cumulants for three choices of rapidity bin from ALICE and two versions of PYTHIA 8.142 at 0.9 TeV.

η range	c_q	ALICE	P8.142	P8.142/135	K_q	ALICE	P8.142	P8.142/135
$ \eta < 0.5$	c_2	1.96(1)(6)	1.73	1.85	K_2	0.68	0.45	0.59
$ \eta < 1.0$	c_2	1.77(1)(4)	1.56	1.70	K_2	0.63	0.42	0.57
$ \eta < 1.3$	c_2	1.70(3)(7)	1.51	1.65	K_2	0.60	0.40	0.55
$ \eta < 0.5$	c_3	5.35(6)(31)	4.16	4.93	K_3	0.82	0.50	0.85
$ \eta < 1.0$	c_3	4.25(3)(20)	3.29	4.11	K_3	0.66	0.42	0.78
$ \eta < 1.3$	c_3	3.91(10)(15)	3.04	3.84	K_3	0.62	0.38	0.73
$ \eta < 0.5$	c_4	18.3(4)(1.6)	12.7	16.8	K_4	1.13	0.70	1.39
$ \eta < 1.0$	c_4	12.6(1)(9)	8.65	12.6	K_4	0.82	0.50	1.24
$ \eta < 1.3$	c_4	10.9(4)(6)	7.60	11.3	K_4	0.57	0.43	1.10

Table 4: Scaled moments and factorial cumulants for three choices of rapidity bin from ALICE and two versions of PYTHIA 8.142 at 2.36 TeV.

η range	c_q	ALICE	P8.142	P8.142/135	K_q	ALICE	P8.142	P8.142/135
$ \eta < 0.5$	c_2	2.02(1)(4)	1.75	1.90	K_2	0.80	0.53	0.69
$ \eta < 1.0$	c_2	1.84(1)(6)	1.61	1.76	K_2	0.73	0.50	0.65
$ \eta < 1.3$	c_2	1.79(3)(7)	1.56	1.71	K_2	0.71	0.48	0.64
$ \eta < 0.5$	c_3	5.76(9)(26)	4.31	5.25	K_3	1.12	0.64	1.08
$ \eta < 1.0$	c_3	4.65(6)(30)	3.56	4.46	K_3	0.88	0.55	0.98
$ \eta < 1.3$	c_3	4.35(16)(33)	3.32	4.23	K_3	0.79	0.51	0.93
$ \eta < 0.5$	c_4	20.6(6)(1.4)	13.4	18.6	K_4	1.77	0.94	2.00
$ \eta < 1.0$	c_4	14.3(3)(1.4)	9.80	14.4	K_4	0.98	0.71	1.68
$ \eta < 1.3$	c_4	12.8(7)(1.5)	8.75	13.1	K_4	0.83	0.61	1.51

These data show a few simple regularities:

1. The values of the c_q moments increase with the value of q and with energy, but decrease with the increasing pseudorapidity bin width. The average multiplicity, as expected, increases with the bin width and energy.

2. The regularities listed above hold for $q = 4$ even in the cases, when the experimental errors given by the authors exceed the differences between the data for different energies or different bin widths. This is not so surprising for the bin width dependence, as the data are here clearly quite strongly correlated. The presence of a similar effect in the energy dependence seems to suggest that the systematic errors at two energies are also correlated.
3. The values of c_q in the two tunings differ by 0.15, 0.9 and 5 for $q = 2, 3$ and 4 , respectively. In contrast, the values of K_q differ much less for $q > 2$. The model with the "wrong" tuning is compatible with data for $q > 2$. This suggests that by a more refined tuning one may get a reasonable agreement with data not only for average multiplicities, but also for higher moments. Using the factorial cumulants we find a smaller spread of the values both in the model and in the data.
4. The moments of the multiplicity distributions are systematically underestimated in the default tuning of PYTHIA 8.142. For the c_q moments the difference between the data and the model values is around 0.2, 1 and 5 for $q = 2, 3$ and 4 , respectively, and increases weakly with the energy. For the K_q moments the trend is the same, but the differences for $q > 2$ are much smaller: only in one case the difference is bigger than 0.5. For the 8.135 tuning the situation is much more involved. All the values of the moments are now significantly higher. Whereas for c_q they are still lower than in the data, the difference is really significant only for $q = 2$. For the K_q the values are below the data for $q = 2$, and above the data for $q = 4$.

It would be highly desirable to calculate reliably the experimental errors of the scaled factorial cumulants to see how significant is the difference between the model and data seen in Tables 3 and 4. We have checked that the statistical uncertainties of the model results for a given set of parameters are negligible: by increasing the statistics by a factor of ten we do not change the values from the Tables by more than a few percent. In all cases the observed fluctuations for c_q are negligible compared with the experimental errors. However, as noted above, tuning the model parameters allows to change the results sufficiently to hope for the agreement with data.

4 Energy dependence from SPS to LHC

The CMS collaboration has also measured the multiplicity distributions for non-single-diffractive (NSD) events at the CM energies of 0.9, 2.36 and 7 TeV [6]. Contrary to the most of published results, where the "NSD" events are defined just by giving the trigger conditions, the CMS data are extrapolated and corrected to remove single diffraction. Since we are unable to repeat this procedure in detail, the precise comparison with MC results is beyond our ability. We have to rely on the effectiveness of the CMS procedure and to compare their data with the events generated in PYTHIA as non-single-diffractive.

The CMS paper refers also to the published data from the lower energies: the CERN collider data from UA5 [7] and UA1 [8] collaborations and the SPS data from the EHS/NA22 collaboration [9]. Thus it is possible to check if the energy evolution in the wider range is reasonably described by the PYTHIA 8 generator, and what are the differences between the different tunings in this range. For the sake of transparency, the UA1 data are shown only in the figure containing the average multiplicities.

In Fig.3 we show the data for the average multiplicity in the range $|\eta| < 2.4$ (2.5 for lower energies) and the PYTHIA 8.142 results with two tunings described above. At the

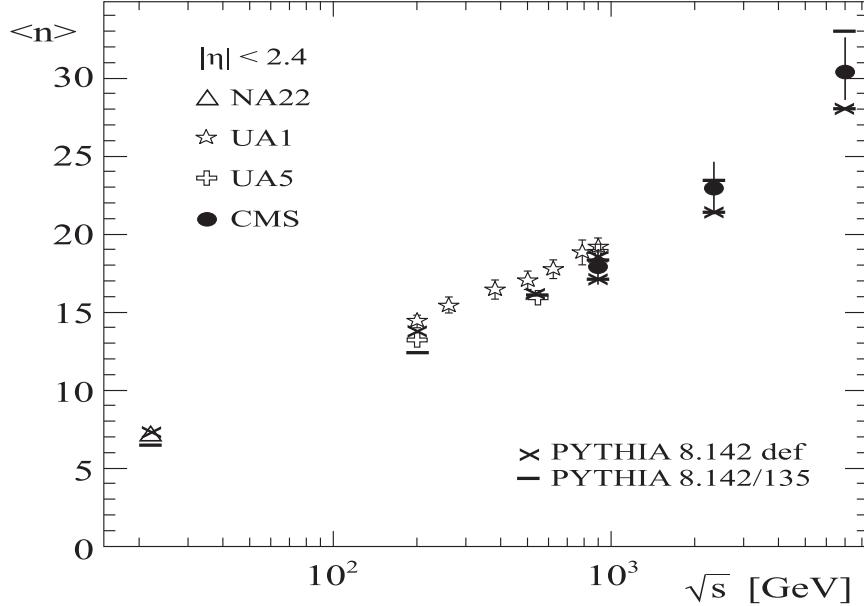


Figure 3: The average multiplicity in the $|\eta| < 2.4$ range from the NA22 data (triangle), UA5 data (open stars), UA1 data (open crosses) and CMS data (black dots). PYTHIA 8.142 predictions for NA22, UA5 and CMS with default and 8.135 tunings are shown as x-s and bars.

CERN collider energies the PYTHIA predictions are shown only with the UA5 trigger conditions. We see that the MC results agree quite well with the observed trend of the data and the values from the two tunings bracket the experimental results.

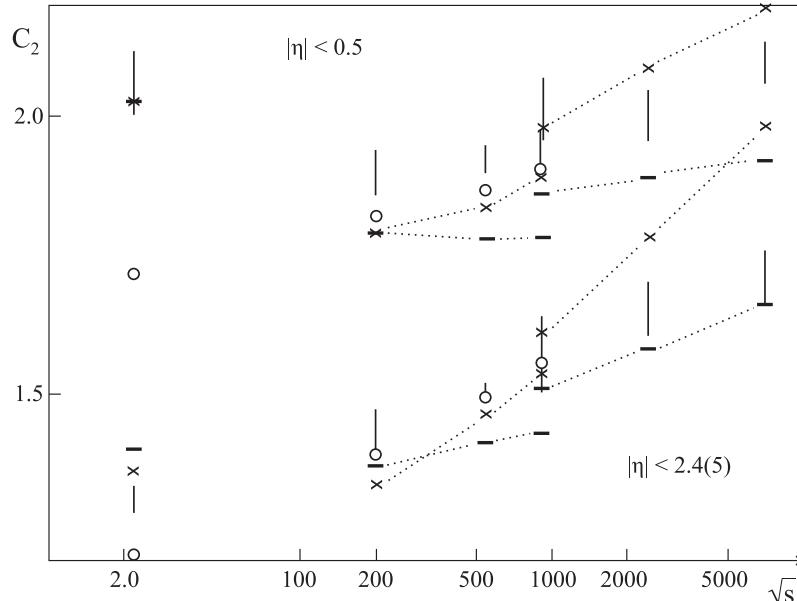


Figure 4: The c_2 moment in the $|\eta| < 0.5$ and $|\eta| < 2.4$ range from the NA22, UA5 and CMS data (vertical bars). PYTHIA 8.142 predictions with default and 8.135 tunings are shown as x-s and bars, respectively. The points for UA5 and CMS data are connected by dotted lines to guide the eye. The open points show the PYTHIA predictions for the pure NSD sample at the UA5 and NA22 energies.

The situation is much more involved for the moments. In Fig.4 we show the second scaled moment c_2 for two choices of the rapidity range: $|\eta| < 0.5$ and $|\eta| < 2.4$ (2.5 for lower energies). Apart from the data and MC results for two tunings (which coincide practically for lower energies) we show here the MC predictions for "true NSD" events. As

shown in Fig.4, the MC predictions for the NA22 data (defined by their trigger) are quite far from the MC results for "true NSD" events. This casts some doubts on the claims of selecting the "true NSD" events from the data by triggers and corrections. For UA5 data the difference is smaller. Note that the difference between the PYTHIA predictions for UA5 and CMS at the same energy of 0.9 TeV results from the different definition of NSD events.

The agreement of the model with data is unsatisfactory: the moments are underestimated at NA22 and UA5 energies for both η ranges. However, the differences are not too big and the energy dependence is qualitatively correct. The most important feature of the results is the sudden increase of differences between the results from two tunings at highest energies. This suggests that the reliable measurement of the multiplicity distribution at 7 TeV should fix the tuning well enough to allow for a significant test of the model from other data.

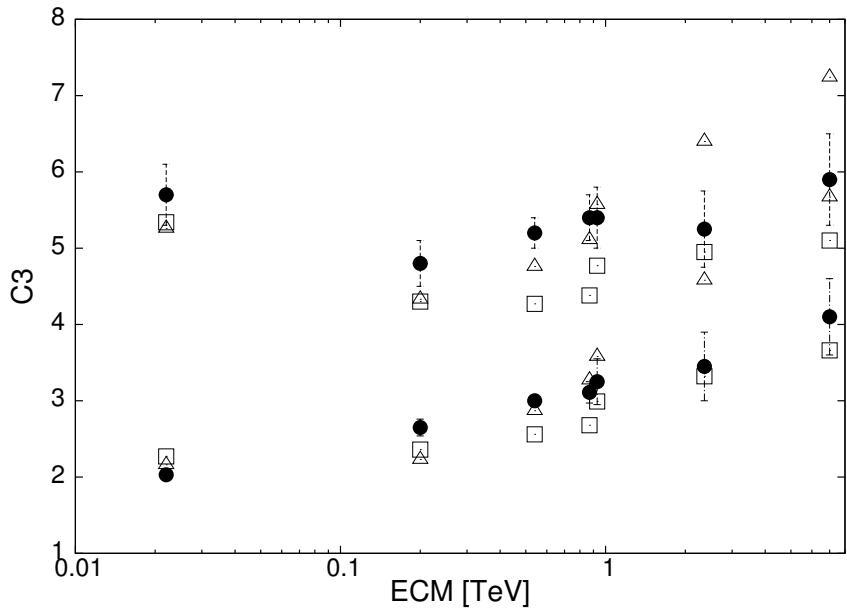


Figure 5: The c_3 moment in the $|\eta| < 0.5$ and the $|\eta| < 2.4$ range from the NA22, UA5 and CMS data (full circles with error bars). PYTHIA 8.142 predictions with default and 8.135 tunings are shown as open squares and triangles, respectively. For transparency, the data points for UA5 and CMS and the PYTHIA predictions at 0.9 TeV are slightly shifted to lower and higher energy, respectively.

In Fig.5 we show the third scaled moment for the rapidity ranges $|\eta| < 0.5$ and $|\eta| < 2.4$ (2.5 for lower energies). We see that the pattern is similar to that of the second moment. For the NA22 data the value of c_3 is again much higher than from the smooth extrapolation of the higher energy data, suggesting that the trigger does not select well the NSD events. The agreement of PYTHIA with low energy data is not quite satisfactory, but the energy dependence is qualitatively correct. The LHC data are bracketed by two versions of PYTHIA 8.142 tunings, which differ strongly at highest energies.

In Fig.6 the fourth scaled moment is shown for the rapidity ranges $|\eta| < 0.5$ and $|\eta| < 2.4$ (2.5 for lower energies), respectively. Again, the pattern is similar. Note that the relative experimental uncertainties are almost the same. This makes the discrepancies at lower energies less significant.

In general, the qualitative agreement of PYTHIA with the energy dependence of the multiplicity distributions should be regarded as acceptable.

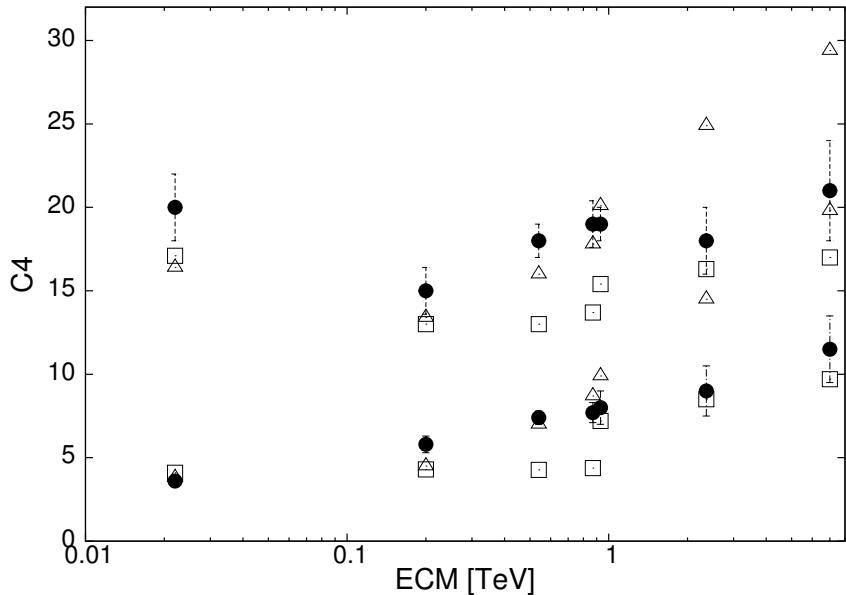


Figure 6: The c_4 moment in the $|\eta| < 0.5$ and the $|\eta| < 2.4$ range from the NA22, UA5 and CMS data (full circles with error bars). PYTHIA 8.142 predictions with default and 8.135 tunings are shown as open squares and triangles, respectively. For transparency, the data points for UA5 and CMS and the PYTHIA predictions at 0.9 TeV are slightly shifted to lower and higher energy, respectively.

5 Conclusions and outlook

We have extended our former analysis of the multiplicity distributions at the LHC energies [1] using the new version of MC generator (PYTHIA 8.142 with two tunings) and calculated the scaled factorial cumulants from the scaled moments and average multiplicities for the model and ALICE data. We have also investigated the energy dependence of the average multiplicity and the three lowest scaled moments in the wide energy range, comparing the PYTHIA results with the NA22, UA5 and CMS data.

We have found that the fast increase of the central density of charged hadrons at LHC energies agrees quite well with the model predictions. The use of factorial cumulants should facilitate the fixing of the tuning parameters. The energy dependence of the scaled moments is qualitatively well described in the wide range covering the SPS, CERN collider and LHC energies.

There is a large difference between the PYTHIA results with two tunings at the highest energies. This suggests that the multiplicity distributions from LHC are well suited to fix the tuning of MC generators. Other data could be then compared with the model predictions.

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